90646

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2003

90646 Use probability distribution models to solve straightforward problems

Credits: Four

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–5 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only		
Achievement Criteria		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Use probability distribution models to solve straightforward problems.	Use probability distribution models to solve problems.	Use and justify probability distribution models to solve complex problems.
Overall Level of Performance		

You are advised to spend 40 minutes answering the questions in this booklet.

Show ALL working.

QUESTION ONE

A student at a hostel finds that the number of letters she receives varies from week to week. Suppose that the number of letters she receives each week can be modelled by a random variable having a Poisson distribution with a mean of two.

Calculate the probability that she receives at most two letters in a particular week.

QUESTION TWO

Polystyrene spheres used for boating are produced with diameters that may be taken to be approximately normally distributed.

A random sample of spheres has a mean diameter 151.3 mm and standard deviation 2.4 mm.

(a) What proportion of spheres has a diameter between 150 and 155 mm?

(b) The 10% of spheres with the largest diameter are classified as being oversize.

What is the minimum diameter of spheres that are oversize?

(c) Spheres are packed as a single row in boxes 920 mm long.

What is the probability that six spheres chosen at random will not fit in a box?

QUESTION THREE

A cardboard carton is filled with 24 blocks of cheese, which are independently obtained from a normally distributed population having mean 820 g and standard deviation 4.5 g.

Empty cardboard cartons are independently obtained from a normally distributed population having mean 270 g and standard deviation 5.1 g.

What is the probability that the total weight of the carton with the 24 blocks of cheese is less than 20 000 g?

QUESTION FOUR

Over the basketball season, Sean scores 4 out of every 10 attempts at a penalty shot from the free throw line.

(a) Calculate the probability he will score at least six times from his next eight throws from the free throw line.

(b) Calculate the probability he will score more than fifty times in his next one hundred throws from the free throw line. Fully justify your method.

Assessment Schedule (sample)

Statistics and Modelling: Use probability distribution models to solve straightforward problems (90646)

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT	Use probability distribution models to solve straight-forward problems.	1	Poisson distribution P $(x \le 2; \lambda = 2)$ $= 0.1353 + 0.2707 + 0.2707$ $= 0.677$	A	Or equivalent	***Achievement:*** Two of Code A No repeated distributions allowed as evidence
		2 (a)	Normal distribution P $(150 < x < 155)$ $=$ P $(-0.542 < z < 1.542)$ $= 0.2061 + 0.4384$ $= 0.645$	A	Or equivalent	
		4 (a)	Binomial distribution P $(x \ge 6; n = 8, \pi = 0.4)$ $= 0.0413 + 0.0079 + 0.0007$ $= 0.050$	A	Or equivalent	
ACHIEVEMENT WITH MERIT	Use probability distribution models to solve problems.	2 (b)	Normal distribution $z = 1.281$ $x = 151.3 + 2.4 \times 1.281$ $= 154.3$ (or 154.4) mm	A M	Or equivalent	***Merit:*** Achievement **plus** Two of Code M **or** Three of Code M
		2 (c)	$E(T) = 6 \times 151.3 = 907.8$ $\sigma(T) = \sqrt{6 \times 2.4^2} = 5.879$ Normal distribution P $(T > 920)$ $=$ P $(z > 2.075)$ $= 0.5 - 0.4810$ $= 0.019$	A M	Or equivalent	
		3	$E(T) = 24 \times 820 + 270 = 19\,950$ $\sigma(T) = \sqrt{24 \times 4.5^2 + 5.1^2}$ $= 22.628$ Normal distribution P $(T < 20\,000)$ $=$ P $(z < 2.210)$ $= 0.5 + 0.4864$ $= 0.986$	A M	Or equivalent	

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH EXCELLENCE	Use and justify probability distribution models to solve complex problems.	4(b)	Normal approximation to binomial appropriate because $n\pi = 40$ and $n(1-\pi) = 60$ are both ≥ 5. Must use a continuity correction. Binomial distribution $P(\mathbf{x} > 50;\ n = 100,\ \pi = 0.4)$ $= P(\mathbf{x} > 50.5)$ normal with continuity correction $= P\left(z > \frac{50.5 - 40}{\sqrt{100 \times 0.4 \times 0.6}}\right)$ $= P(z > 2.143)$ $= 0.5 - 0.4839$ $= 0.016$	A M E	Without continuity correction. Or equivalent	***Excellence:*** Merit **plus** Code E